Exercise 1. Evaluate:

(a) $\int t^4 + 3t^2 + 2t + \pi dt$

(b) $\int \sin x + 3e^x dx$

(c) $\int (x+2)(x-3)dx$

(d) $\int \frac{t\sin t - 2t^{3/5} + \pi t^{\pi+1}}{t} dt$

(e) (Challenge) $\int 3e^x + \cos(2x)dx$

Solutions:

(a) First, apply linearity to get that

$$\int t^4 + 3t^2 + 2t\pi dt = \int t^4 dt + 3\int t^2 dt + 2\int t dt + \int \pi dt.$$

Next, we apply the anti-power rule to get:

$$\int t^4 + 3t^2 + 2t\pi dt = \int t^4 dt + 3\int t^2 dt + 2\int t dt + \int \pi dt = \boxed{\frac{t^5}{5} + 3\frac{t^3}{3} + 2\frac{t^2}{2} + \pi t + C.}$$

(b) We apply linearity to get

$$\int \sin x + 3e^x dx = \int \sin x dx + 3\int e^x dx = \boxed{-\cos x + 3e^x + C.}$$

(c) First, we expand the product then apply linearity and the anti-power rule to get:

$$\int (x+2)(x-3)dx = \int x^2 - x - 6dx = \int x^2 dx - \int x dx - 6\int dx = \boxed{\frac{x^3}{3} - \frac{x^2}{2} - 6x + C.}$$

(d) First, we simplify the integrand then apply linearity and the anti-power rule to get:

$$\begin{aligned}\int \frac{t\sin t - 2t^{3/5} + \pi t^{\pi+1}}{t} dt &= \int \sin t - 2t^{3/5-1} + \pi t^{\pi} dt \\ &= \int \sin t dt - 2\int t^{3/5-1} dt + \pi \int t^{\pi} dt \\ &= \boxed{-\cos t - 2\left(\frac{t^{3/5}}{3/5}\right) + \pi\left(\frac{t^{\pi+1}}{\pi+1}\right) + C.}\end{aligned}$$

(e) First, we apply linearity to get

$$\int 3e^x + \cos(2x)dx = 3\int e^x dx + \int \cos(2x)dx = 3e^x + \int \cos(2x)dx.$$

Naively, we'd hope that $\sin(2x)$ is an antiderivative of $\cos(2x)$ but

$$\frac{d}{dx}\sin(2x) = 2\cos(2x)$$

by the chain rule. However, if we divide by 2 then we see that

$$\frac{d}{dx}\frac{\sin(2x)}{2} = \frac{2\cos(2x)}{2} = \cos(2x).$$

Therefore, we have

$$\int\int 3e^x + \cos(2x)dx = 3\int e^x dx + \int \cos(2x)dx = \boxed{3e^x + \frac{1}{2}\sin(2x) + C.}$$

Exercise 2. Which of the following are antiderivatives of $f(x) = 2x + 1$:

(a) $F(x) = 2x^2 + x + \pi$

(b) $G(x) = (x + 1/2)^2$

(c) $H(x) = (x + e)^2$

(d) $I(x) = x^2 + x - 4$

Solutions:

(a) Note

$$F'(x) = 4x + 1 \neq f(x)$$

so $\boxed{F(x) \text{ is not.}}$

(b) Note

$$G'(x) = 2(x + 1/2) = 2x + 1 = f(x)$$

so $\boxed{G(x) \text{ is.}}$

(c) Note

$$H'(x) = 2(x + e) = 2x + 2e \neq f(x)$$

so $\boxed{H(x) \text{ is not.}}$

(d) Note

$$I'(X) = 2x + 1 = f(x)$$

so $\boxed{I(X) \text{ is.}}$