Exercise 1. (Trigonometry)

(a) Find all solutions in $[0, 2\pi]$ to $\cos(2\theta) = \sin\theta$.

(b) Compute the exact value of $\sin\left(\frac{11\pi}{12}\right)$.

(c) Compute the exact value of $\sin^{-1}(-1/2)$.

Exercise 2. (One-sided limits)
Find all values, if they exist, of k which make $f(x)$ continuous and k which make $f(x)$ differentiable.

(a) Let

$$f(x) = \begin{cases} k\sqrt{x+4} + k^2 & x \geq 0 \\ k^5 x - 1 & x < 0 \end{cases}.$$

(b) Let

$$f(x) = \begin{cases} kx + 1 & x \geq k \\ 3x + 2 & x < k \end{cases}.$$

Exercise 3. (Limits)
Compute the following limits:

(a)

$$\lim_{x\to 1} \frac{x^3 - 1}{x - 1}$$

(b)

$$\lim_{x\to 2} \frac{\sqrt{x+2} - 2}{x - 2}$$

(c)

$$\lim_{x\to\infty} \frac{x^2 - 9x + 2}{3x^2 - 7x + 1}$$

(d)

$$\lim_{x\to\infty} \frac{\ln x}{x}$$

Exercise 4. (Derivatives)
Compute the derivative of:

(a)

$$f(x) = \frac{\tan(e^x)}{2x^2 + 1}.$$

(b)

$$f(x) = (\sin 2x)^{\sqrt{3x}}.$$

Exercise 5. (Rate of change)
A particle moves along a line with position $s(t) = \frac{1}{3}t^3 - 2t^2 + 3$ (meters) at time t (seconds).

(a) Find the velocity function.

(b) At what time t is the particle at rest?

(c) On which time intervals, is the particle speeding up? Speeding down?

Exercise 6. (Tangent lines)
Let

$$f(x) = e^{2x} \sin 2x, \quad 0 \leq x \leq \pi.$$

Find all $x \in [0, \pi]$ on the graph of $f(x)$ where the tangent line has slope 0 and the equation of these tangent lines.

Exercise 7. (Implicit differentiation)
Compute $\frac{dy}{dx}$ for

$$3xy - xy^3 - 2 = 0.$$

Find all points on the graph of the above equation whose tangent line has an infinite slope. Find the tangent line of the above equation at the point $(1, 2)$.

Exercise 8. (Derivatives of inverse functions)
Compute $(f^{-1})'(a)$ for the given $f(x)$ and a:

(a) $f(x) = x^3 + 2x - 3$ with $a = 0$.

(b) $f(x) = x - \frac{2}{x}$, for $x > 0$, with $a = 1$.

Exercise 9. (Linear approximation)
Use linear approximations to approximate the following values with the given function and point, a.

(a) $\sqrt{16 + 1/100}$ with $f(x) = \sqrt{x}$ and $a = 16$.

(b) $\sin\left(\frac{314}{100}\right)$ with $f(x) = \sin x$ and $a = \pi$.

Exercise 10. (Minima/Maxima)
Find the minima and maxima of the given functions over the given intervals:

(a) $f(x) = x \sin x + \cos x$ where $x \in [0, 2\pi]$.

(b) $f(x) = x^x$ where $x \in (0, \infty)$.

Exercise 11. (Mean Value Theorem)

(a) Use the mean value theorem to show that $e^x - e$ has exactly one root.

(b) Determine whether the mean value theorem applies to

$$f(x) = \ln(2x - 1), \quad 1 \leq x \leq 3.$$

If the mean value theorem applies, find all $c \in (1, 3)$ such that

$$(3 - 1)f'(c) = f(3) - f(1).$$

(*What guarantees that such a c exists?*)